Reviving and Extending the Mid-Field Phase Screen Model for Distributed Aberration Correction

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MOTIVATION + BACKGROUND

- Several clinical applications of medical pulse-echo ultrasound imaging benefit from aberration correction

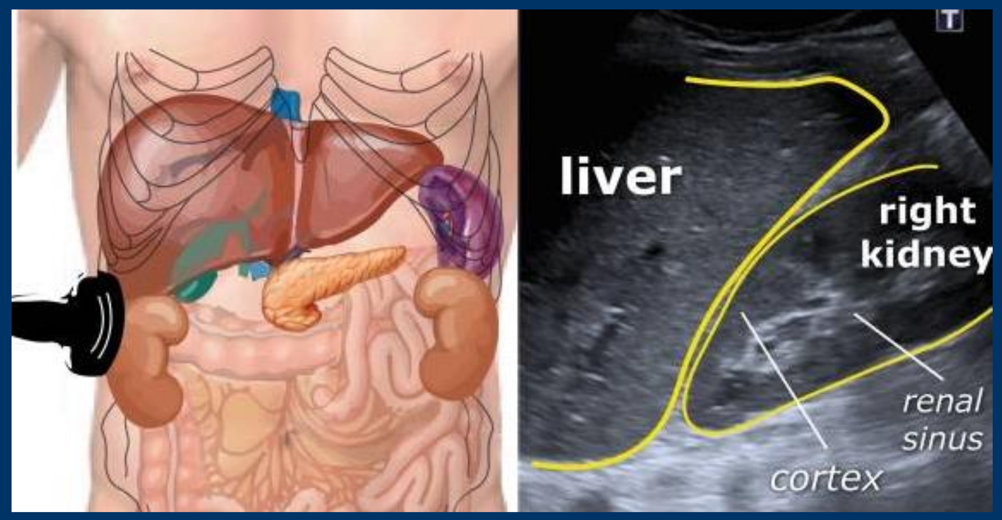

Figure 1. Handheld Ultrasound for Abdominal Imaging

- In abdominal imaging (Figure 1), aberration correction is used (a) to estimate sound speed as a quantitative measure of liver fat fraction[2] and (b) correct aberrations in kidney images[3].

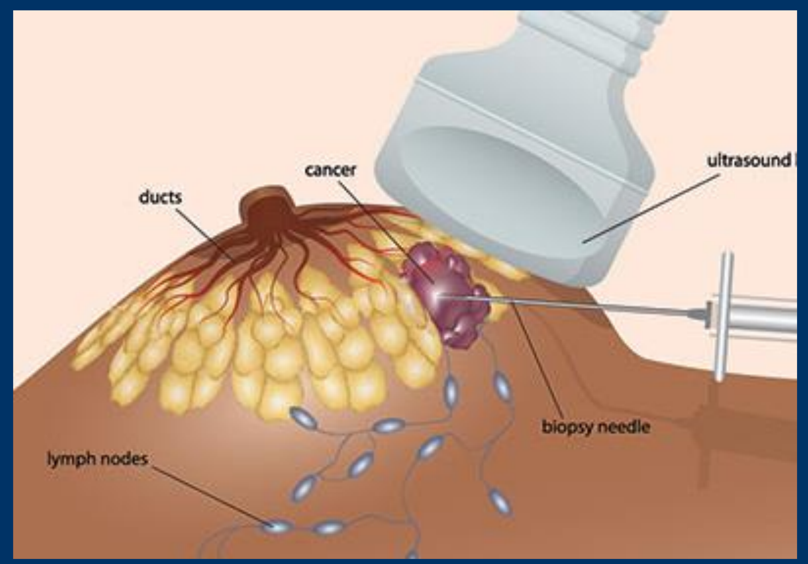

Figure 2. Handheld Ultrasound of the Breast

- Aberrations in breast imaging (Figure 2) can be used to reconstruct and correct for local variations in sound speed indicative of cancerous masses[4].

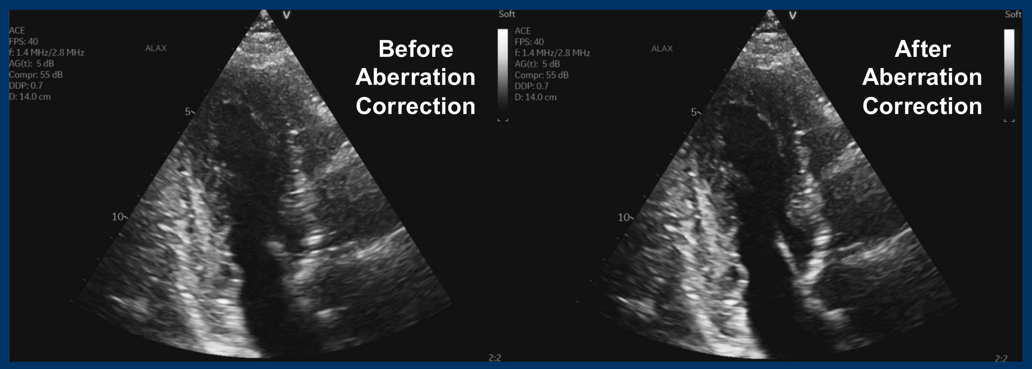

Figure 3. Aberration Correction in Cardiac Ultrasound[1]

- Even in imaging applications (such as cardiac imaging[1]) where sound speed estimation is not the main focus, aberration correction can make anatomical features clearer (Figure 2).

MID-FIELD PHASE SCREEN MODEL

- Physics Background: All aberrations in medical ultrasound imaging are caused by sound speed variations and deviations from the traditionally assumed tissue sound speed (1540 m/s)
- Near-Field Phase Screen: Single set of aberration delays/phases at transducer surface[5]
  - Does not capture spatial variation in aberration delay caused by a full sound speed profile

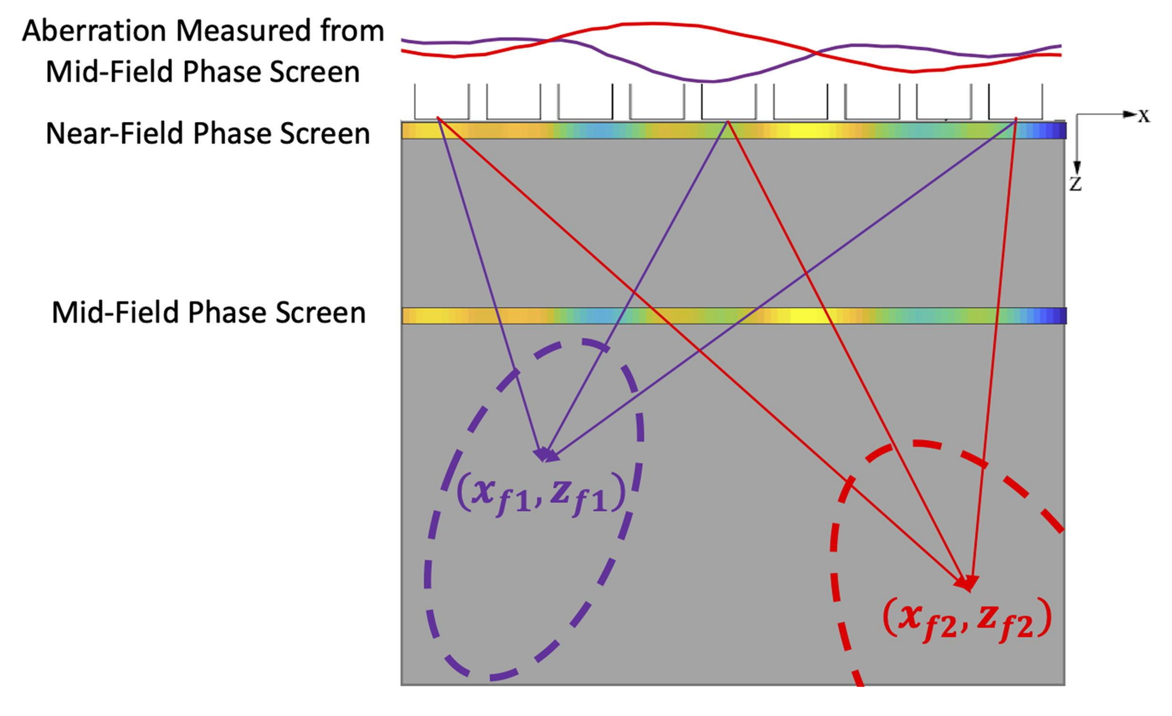

Figure 4. Parallax Effect Induced by Mid-Field phase screen.

- Mid-Field Phase Screen: Offset between aberrating screen and transducer surface[6]
  - Captures spatial variation in aberration delays (finite *isoplanatic patches*—see Figure 4) caused by *parallax* induced at a *specific depth*

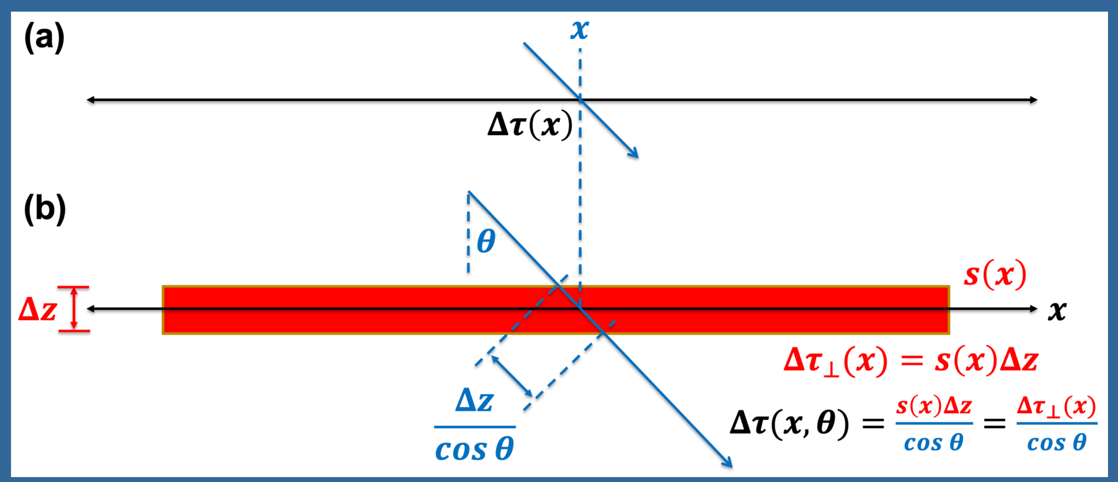

Figure 5. Phase Screen Models. (a) Original Formulation[5,6]. (b) Revised Formulation Considering the Angle of Intersection.

- Traditional phase screen models[5,6] parameterized by fixed aberration delays $\Delta\tau(x)$ over the screen
  - Ignores angle of incidence (Figure 5a)
- Thin slab of thickness Δz with slowness $s(x)$
  - Delays at normal incidence: $\Delta\tau_\perp(x) = s(x)\Delta z$
  - With correct angular dependence (Figure 5b):
    - $\Delta\tau(x,\theta) = \frac{s(x)\Delta z}{\cos\theta} = \frac{\Delta\tau_\perp(x)}{\cos\theta}$
    - *extends to infinitesimally thin screen*
    - correct physics + no added computation cost

MULTIPLE MID-FIELD PHASE SCREENS

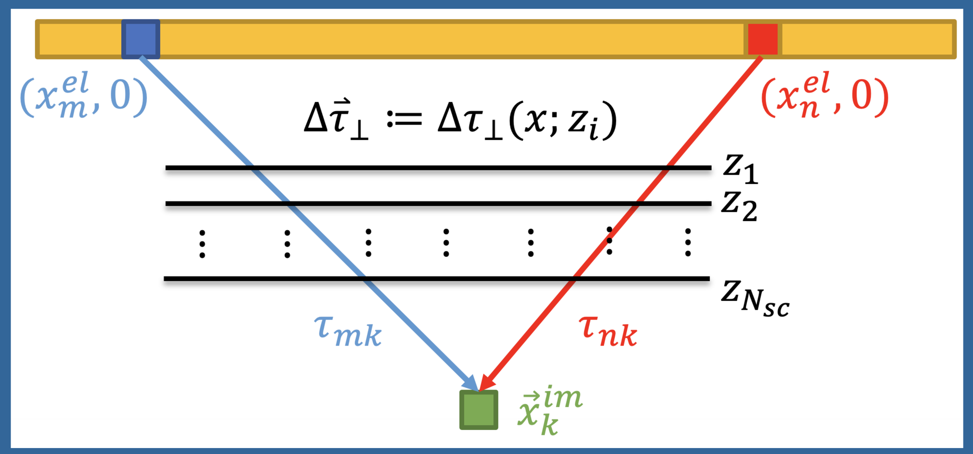

Figure 6. Visualization of $\nabla_s E$ the backprojection of the error $\boldsymbol{p}_i - \boldsymbol{p}_{obs,i}$ onto slowness profile during waveform inversion

- Need mid-field phase screens $\Delta\tau_\perp(x; z_i)$ at multiple depths z_i to capture aberrations caused by full spatial variation in sound speed (Figure 6)
  - Can convert $\Delta\tau_\perp(x; z_i)$ to equivalent slowness $s(x, z)$ and/or sound speed $c(x, z)$ profile
  - Faster/easier than modeling path-length matrix
  - Simplified approach that neglects refraction
  - Flexibility over spacing and number of screens

DIFFERENTIABLE BEAMFORMING MODEL

- Multistatic synthetic aperture dataset $\tilde{p}_{mn}(t)$
  - full matrix of transmitters m and receivers n
  - signal traces as a function of time t
- Time of flight τ_{lk} from element l to image point $\vec{x}_k^{im}$
- Applying focusing **delays** (according to Figure 6):

$$p_{mn}(\vec{x}_k^{im}; \Delta\vec{\tau}_\perp) = \tilde{p}_{mn}(\tau_{mk}(\Delta\vec{\tau}_\perp) + \tau_{nk}(\Delta\vec{\tau}_\perp))$$

- **Summation** over receivers:

$$I_m(\vec{x}_k^{im}; \Delta\vec{\tau}_\perp) = \sum_{n=1}^{N} p_{mn}(\vec{x}_k^{im}; \Delta\vec{\tau}_\perp)$$

- Direct Least-Squares Aberration Correction:

$$E(\Delta\vec{\tau}_\perp) = \frac{1}{2}\sum_{k}\sum_{m=1}^{N-1}\left|I_{m+1}(\vec{x}_k^{im}; \Delta\vec{\tau}_\perp) - I_m(\vec{x}_k^{im}; \Delta\vec{\tau}_\perp)\right|^2$$

DEMONSTRATION IN K-WAVE SIMULATION

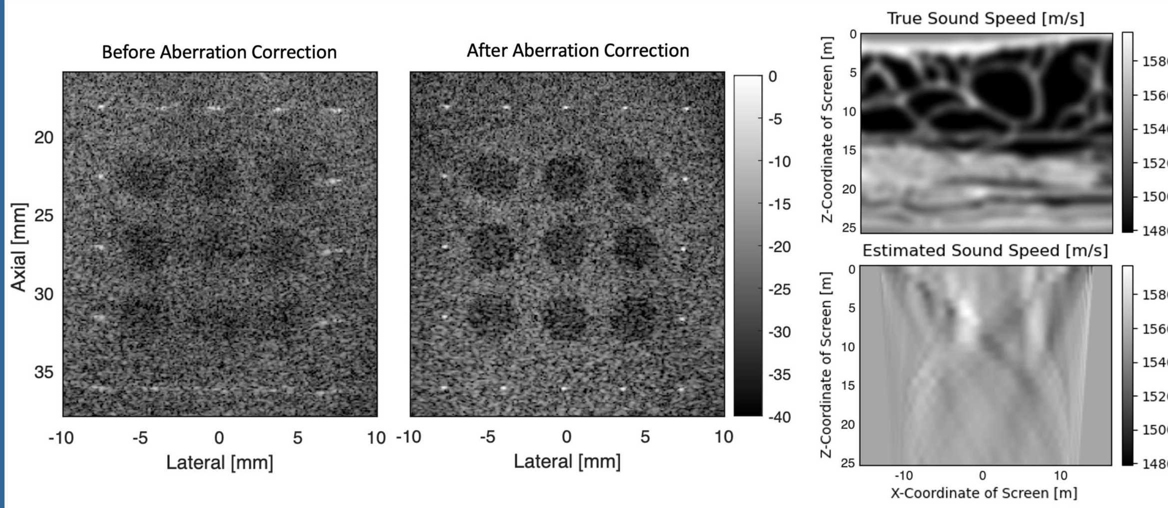

Figure 7. Sound speed estimation and aberration correction using multiple mid-field screen in k-Wave simulation[8,9]. The full-width-at-half-maximum resolution improves from 2.19 ± 0.92 mm to 0.31 ± 0.03 mm.

DEMONSTRATION ON IN-VITRO PHANTOM EXPERIMENTS

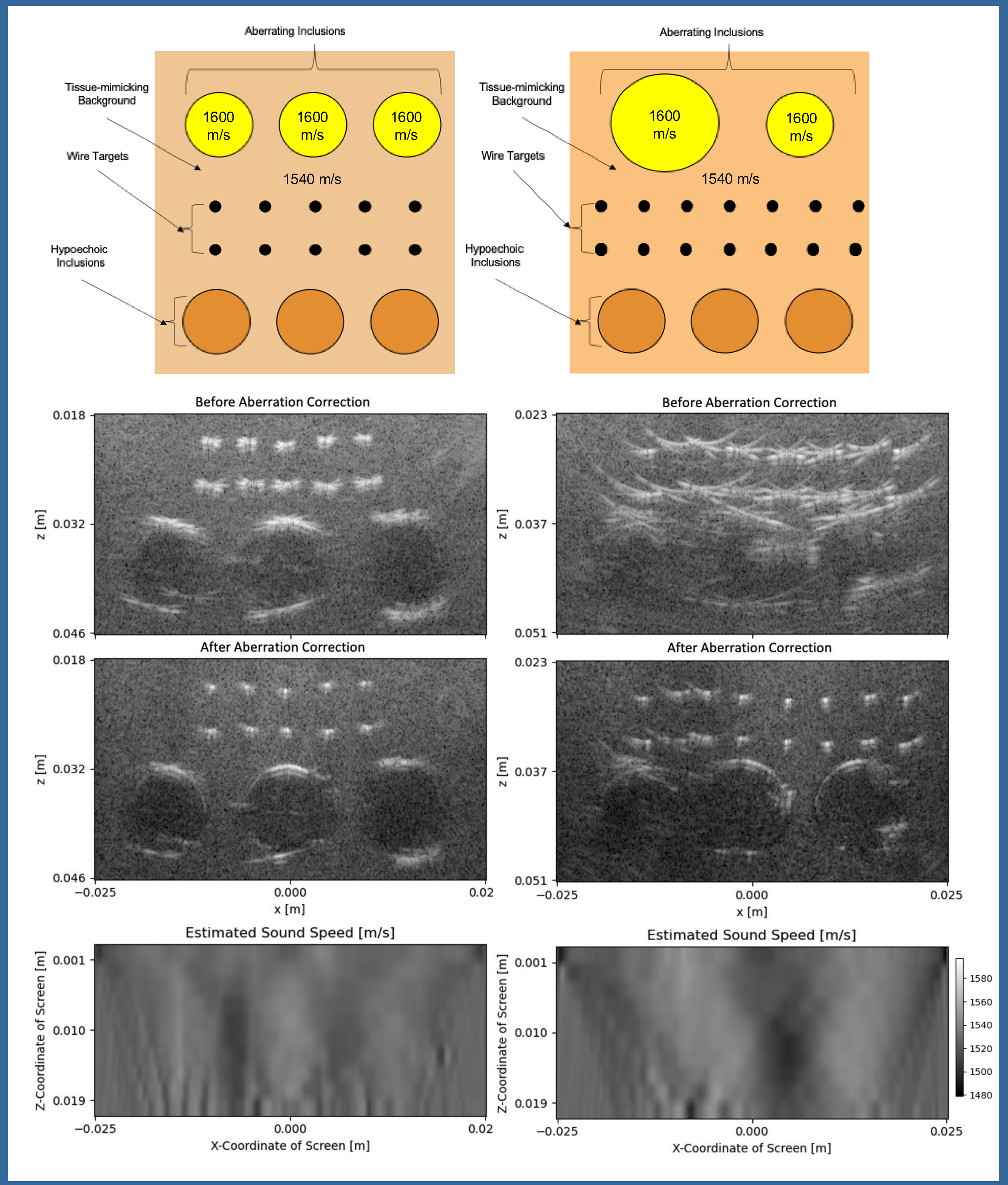

Figure 8. In-vitro phantom experiments demonstrating sound speed estimation and aberration correction using multiple mid-field phase screens. There is a 61.79 ± 18.97% and a 51.71 ± 17.91% improvement in point target resolution on the left and right, respectively.

CONCLUSIONS

- Multiple mid-field phase screens offer a simplified modeling of aberration delays in spatially varying sound speed medium

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