Medical Pulse-Echo Ultrasound Imaging Based on the Cross-Correlation of Transmitted and Backpropagated-Receive Wavefields

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***Abstract*—In seismic imaging, a shot gather collects time series data from all receivers (geophones) with a common transmitter (source or shot). Shot-gather migration is an imaging technique that performs time-domain cross-correlation of the transmitted wavefield with the back-propagated receive wavefield. Shot-gather migration can be used to achieve the focusing quality of synthetic aperture imaging without susceptibility to motion by accurately modeling each wave-front, regardless of the transmit sequence applied. Since wave propagation is parameterized by the speed of sound in the medium, shot gather migration can also be used to perform distributed phase aberration correction enabling additional focusing improvements. In this work, we illustrate the shot gather migration process for point targets in simulation.**

***Index Terms*—Shot-Gather Migration, Medical Ultrasound Imaging, Adjoint-Based Method**

I. Introduction

Ultrasound imaging typically uses delay-and-sum beamforming to coherently sum backscattered echo signals to obtain B-mode images. The most common transmission scheme in medical ultrasound uses a sequence of focused transmit beams to image the medium because of its ability to maintain high-SNR, especially around the focal zones. Although typical clinical systems using a single transmit per A-line, multiple transmit events could contribute to the image for a particular imaging line, as in virtual source synthetic aperture [1].

The goal of virtual source synthetic aperture is to employ a set of parallel imaging lines for each transmit beam so that multiple transmit beams contribute to a particular A-line. However, because this approach only applies to the main support of the beam, a discontinuity artifact occurs at the focal depth of the transmit beams. Certain approaches have also been proposed to incorporate back-scatter from the edge waves produced by a focused transmit beam in order to mitigate this discontinuity and improve focusing [2].

Another limitation of virtual source synthetic aperture is the assumption that the transmit focus acts as a point source. This is major limitation in the presence of sound speed errors because phase aberration prevents the transmitted sound from converging to a single point at the same time; i.e., the transmit focus can no longer be understood as a point source in presence of phase aberration.

Recent works [3]–[5] attempt to recover the multistatic synthetic aperture dataset from focused transmit beams to circumvent the need to apply virtual sources, improve focusing at all depths, and adjust focusing delays based on the sound speed in the medium. However, a major drawback of these approaches is that it requires at least as many distinct transmit beams as there are elements on the probe to accurately recover the multistatic dataset. However, for certain imaging applications, such a large number of transmit events can impose a serious limitation on frame rate.

Rather than recover the multistatic synthetic aperture dataset, we propose an imaging method does not require any assumed virtual sources, and remains agnostic to the transmit sequence applied. Instead of applying standard delay-and-sum beamforming, we model the wave propagation to the imaging point and implement the adjoint of the backscattering process, which is back-propagation of receive signals into the medium and cross-correlating with the transmitted pulse. In geophysics, this type of imaging is known as shot-gather migration [6]. In this approach, the transmitted pulse acts as a matched filter for the receive signals that are back-propagated into the medium. By accounting for the full-wave nature of ultrasound physics, shot-gather migration only requires knowledge of the transmit pulse, apodization, and delays. Since wave propagation in heterogeneous media can be simulated, this imaging approach can adjust for sound speed heterogeneity without relying on any assumed virtual sources. We illustrate the imaging process for point targets in Field II simulations, and leave the investigations into its implications on virtual source synthetic aperture and focused transmit beams for future work.

II. Theory

A. Forward Model for Wave Propagation

The two-dimensional wave equation can be written as

$$\left(\nabla^2_{x,z} - s^2(x,z)\frac{\partial^2}{\partial t^2}\right)p(x,z,t) = f(x,z,t), \qquad (1)$$

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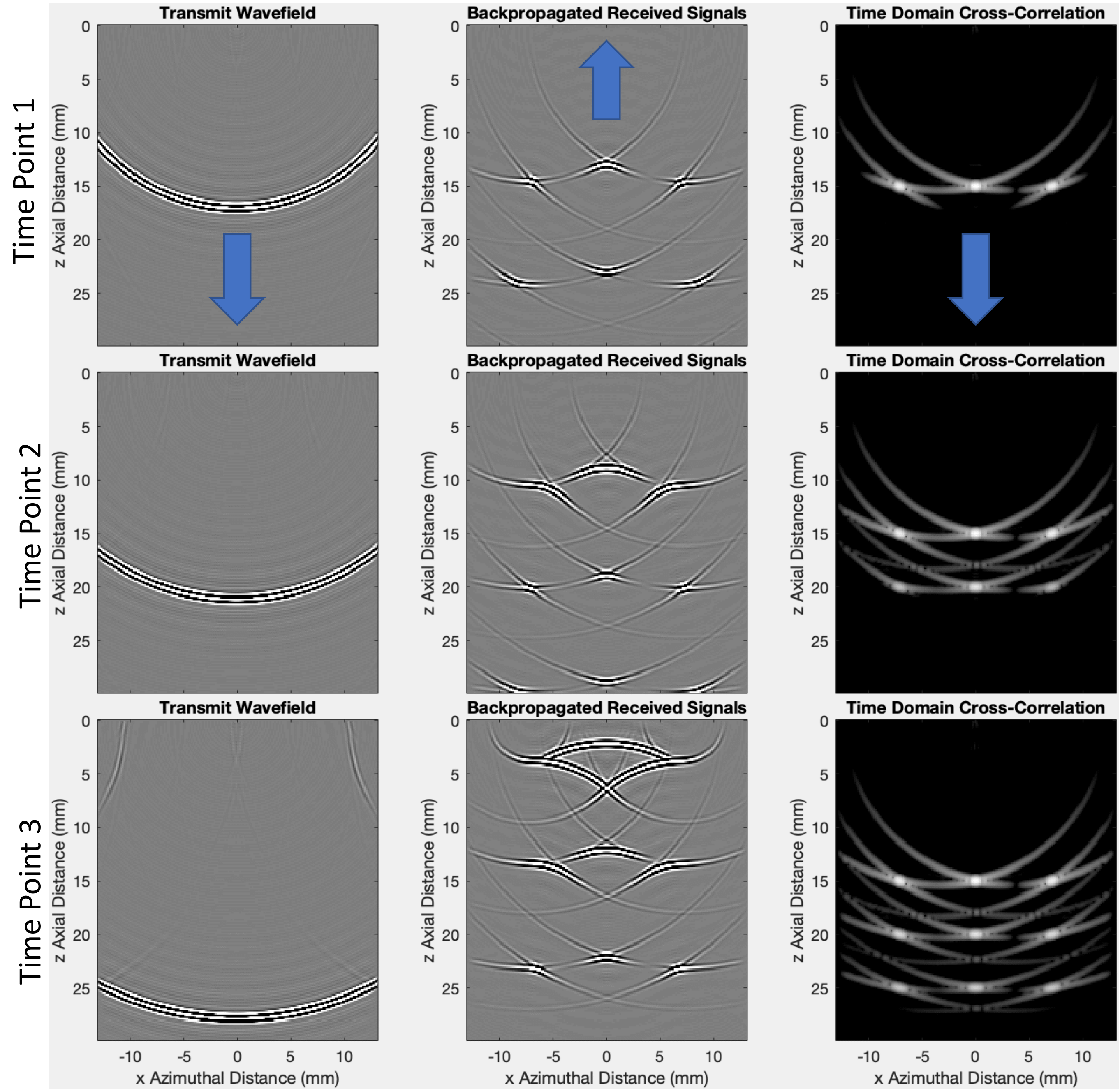

Fig. 1. Demonstration of Time-Domain Cross Correlation of Transmitted and Backpropagated Received Wavefields with Point Targets in Field II Simulations. The receive wavefronts converge on their respective point target locations as the primary transmit wavefield passes over them.

where $s(x,z)$ is the slowness (i.e. reciprocal of the speed of sound) in the medium, $p(x,z,t)$ is the pressure profile in the medium as a function of space (x,z) and time t, and $f(x,z,t)$ is a forcing term that serves to excite the medium (e.g. a transmit pulse from transducer).

Treating the term in parentheses as an operator, we can find $p(x,z,t)$ as

$$p(x,z,t) = \left(\nabla^2_{x,z} - s^2(x,z)\frac{\partial^2}{\partial t^2}\right)^{-1}_{fwd,t} f(x,z,t). \qquad (2)$$

The subscript "fwd, t" indicates that the wave propagation happens in forward time when we solve the PDE because the forcing term $f(x,z,t)$ is actually intended to represent the initial boundary condition.

We denote $p_{rx}(x,z,t)$ as the pressure field recorded by the transducer and $\mathcal{K}$ as a sampling operator that records the pressure at the transducer surface (i.e., $p_{rx}(x,z,t) = \mathcal{K}p(x,z,t)$)

so that

$$p_{rx}(x,z,t) = \mathcal{K}\left(\nabla^2_{x,z} - s^2(x,z)\frac{\partial^2}{\partial t^2}\right)^{-1}_{fwd,t} f(x,z,t). \quad (3)$$

B. Gradient at the First Step of Waveform Inversion

The previous section is a model description for the receive wavefield based on the transmit pulse as a forcing function. Now we will describe the gradient calculation at the first step of waveform inversion, and how this gradient yields an image reconstruction of the back-scatter in the medium. We define the following misfit $J(s^2)$ as a function of the squared slowness s^2:

$$J(s^2) = \frac{1}{2}\int_{\mathbb{R}}\left(\int_0^\infty |p_{rx}^{obs} - p_{rx}(s^2)|^2 dt\right) dx, dz \quad (4)$$

Waveform inversion would involve computing the gradient of this misfit as a function of the squared slowness s^2:

$$\frac{\delta J(s^2)}{\delta s^2} = \int_{\mathbb{R}}\left(\int_0^\infty \left(\frac{\delta p_{rx}}{\delta s^2}\right)^* \left(p_{rx}^{obs} - p_{rx}(s^2)\right) dt\right) dx, dz \quad (5)$$

where * denotes complex conjugation.

Typically, the first step of waveform inversion assumes a constant (or sufficiently smooth) slowness model so that no sound would be backscattered from the simulated medium (i.e. $p_{rx}(x,z,t) = 0$), so that the gradient is:

$$\frac{\delta J(s^2)}{\delta s^2} = \int_{\mathbb{R}}\left(\int_0^\infty \left(\frac{\delta p_{rx}}{\delta s^2}\right)^* p_{rx}^{obs} dt\right) dx, dz \quad (6)$$

Furthermore, we call the term in parentheses as the gradient image $I(x,z)$ so that

$$\frac{\delta J(s^2)}{\delta s^2} = \int_{\mathbb{R}} I(x,z)\, dx, dz \quad (7)$$

$$I(x,z) = \int_0^\infty \left(\frac{\delta p_{rx}}{\delta s^2}\right)^* p_{rx}^{obs} dt \quad (8)$$

Now we show the gradient of the receive signals with respect to the squared slowness:

$$\frac{\delta p_{rx}}{\delta s^2} = \mathcal{K}\frac{\delta\left(\nabla^2 - s^2\frac{\partial^2}{\partial t^2}\right)^{-1}_{fwd,t}}{\delta s^2} f. \quad (9)$$

To compute the gradient of the wave equation solution operator, we review the derivative of an inversion operator as a function of a parameter:

$$\frac{\partial \boldsymbol{A}^{-1}}{\partial \alpha} = -\boldsymbol{A}^{-1}\frac{\partial \boldsymbol{A}}{\partial \alpha}\boldsymbol{A}^{-1} \quad (10)$$

where $\boldsymbol{A}(\alpha)$ is a linear operator and α is a parameter of the linear operator. When this is applied to equation (9), we obtain

$$\begin{aligned}\frac{\delta p_{rx}}{\delta s^2} &= -\mathcal{K}\left(\nabla^2 - s^2\frac{\partial^2}{\partial t^2}\right)^{-1}_{fwd,t} \\ &\quad \frac{\delta\left(\nabla^2 - s^2\frac{\partial^2}{\partial t^2}\right)_{fwd,t}}{\delta s^2}\left(\nabla^2 - s^2\frac{\partial^2}{\partial t^2}\right)^{-1}_{fwd,t} f \\ &= \mathcal{K}\left(\nabla^2 - s^2\frac{\partial^2}{\partial t^2}\right)^{-1}_{fwd,t}\frac{\partial^2}{\partial t^2}\left(\nabla^2 - s^2\frac{\partial^2}{\partial t^2}\right)^{-1}_{fwd,t} f \\ &= \mathcal{K}\left(\nabla^2 - s^2\frac{\partial^2}{\partial t^2}\right)^{-1}_{fwd,t}\frac{\partial^2 p}{\partial t^2},\end{aligned} \quad (11)$$

and the adjoint of this gradient is

$$\left(\frac{\delta p_{rx}}{\delta s^2}\right)^* = \frac{\partial^2 p^*}{\partial t^2}\left(\nabla^2 - s^2\frac{\partial^2}{\partial t^2}\right)^{-1}_{rev,t}\mathcal{K}^*, \quad (12)$$

so that

$$I(x,z) = \int_0^\infty \frac{\partial^2 p^*}{\partial t^2}\left[\left(\nabla^2 - s^2\frac{\partial^2}{\partial t^2}\right)^{-1}_{rev,t}\mathcal{K}^* p_{rx}^{obs}\right] dt. \quad (13)$$

The subscript "rev,t" indicates that although the wave-equation operator is self-adjoint, wave-propagation in the adjoint occurs in reverse-time rather than forward in time. The term in brackets [] is the backpropagated receive wavefield (in reverse time). Recall that p_{rx} is are the receive signals and $\mathcal{K}$ is the sampling operator for recording the pressure field at the transducer surface. The adjoint of the sampling operator $\mathcal{K}^*$ can be interpreted as a reinjection of the receive signals into the medium and the wave equation operator in front of it plays the signals back into the medium in reverse time. This backpropagated receive wavefield (in brackets []) is cross-correlated with the second time-derivative of the transmitted wavefield $\frac{\partial^2 p}{\partial t^2}$.

III. Methods

A. Image Formation Process

Although, we motivate this image formation process through waveform inversion, we choose not apply a second time-derivative to the transmitted wavefield in order to avoid unnecessary noise amplification in the ultrasound image. Thus, the final image is

$$I(x,z) = \left|\int_0^\infty p_{tx}^*(x,z,t)\, p_{rx,back}^{obs}(x,z,t) dt\right|, \quad (14)$$

where $p_{tx}(x,z,t)$ is the transmitted wavefield and $p_{rx,back}^{obs}(x,z,t)$ is the backpropagated receive wavefield. We use the angular spectrum method [7], [8] for wave-propagation.

B. Field II Simulation

Field II was used to simulate the complete multistatic dataset for point targets. The Field II simulation used a 96-element linear array with 0.154 mm pitch, and transmit pulse with 5 MHz center frequency and 70% fractional bandwidth. The sound speed in the medium is 1540 m/s and the sampling frequency of the channel data is 100 MHz.

IV. Results and Discussion

Figure 1 shows the time-domain cross-correlation process for the image reconstruction of point targets. As the primary transmitted wave moves into the medium, the backpropagated receive signals move upward towards the transducer surface. The wavefronts in the receive wavefield converge on point target locations as the primary transmit pulse passes over these locations. This type of image reconstruction process is possible with any imaging target (e.g diffuse scattering, planar reflectors, and point targets) or transmit sequence (e.g. focused-waves, diverging-waves, and plane-waves). It also possible to implement this in the frequency-domain, which may be more computationally efficient if the solution of the wave-equation is based on the Helmholtz equation or the angular spectrum method. The image reconstruction process was developed from the first gradient of the waveform-inversion process used to estimate the slowness in the medium. A potential implication of the work presented here is that this image reconstruction process can be applied iteratively to perform a full-waveform inversion reconstruction of sound speed in medical ultrasound imaging. This full-waveform imaging approach opens new avenues for image reconstruction and parameter estimation because it relies on models of wave propagation rather than simple delay-and-sum.

V. Conclusions and Future Work

We demonstrate initial work on an image formation process based on the time-domain cross-correlation of transmitted and backpropagated receive wavefields. In future work, we hope to demonstrate the frequency-domain equivalent of this imaging process and to thoroughly explore its implications on imaging with focused transmit beams and full-waveform inversion.

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